Finding the Shortest-Path between two Locations

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ABSTRACT

Dijkstra's Shortest-Path Algorithm is presented and considered with examples. Then finding the shortest-path between two locations are studied.

Keywords : weighted graph, label, shortest-path, minimum length.

INTRODUCTION

A graph with numbers on the edges is called a weighted graph. If edge e is labeled k, we say that the weight of edge e is k. In a weighted graph, the length of a path is the sum of weights of the edges in the path. In weighted graph, we let w(i, j) denote the weight of edge (i, j). We often want to find a shortest-path (i.e., a path having minimum length) between given vertices. We assume that the weights are positive numbers and that we want to find a shortest-path, say, from vertices a to vertex z.

Dijkstra's algorithm involves assigning labels to vertices. We let L(v) denote the label of vertex v. At any point, some vertices have temporary labels and the rest have permanent labels.

We let T denote the set of vertices having temporary labels. In illustrating the algorithm, we will circle vertices having permanent labels. Each iteration of the algorithm changes the states of one label from temporary to permanent. Thus we may terminate the algorithm when z receives a permanent label. At this point L(z) gives the length of a shortest path from a to z.

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Dijkstra's Shortest-Path Algorithm

In put : A connected, weighted graph in which all weights are positive.

Vertices a and z.

Out put : L(z), the length of shortest path from a to z.

1. [Initialization] Set L(a) : = 0. For all vertices $x \neq a$, set $L(x) := \infty$.

   Let T be the set of vertices.

2. [Done?] If $z \notin T$, stop.

   (L(z) is the length of shortest path from a to z).

3. [Get next vertex.] Choose $v \in T$ with the smallest value of L(v). Set

   $T := T - \{v\}$.

4. [Revise labels.] For each vertex $x \in T$ adjacent to v, set

   $L(x) := \min\{L(x),\ L(v) + w(v,\ x)\}$. Go to line 2.

Example . We show how the algorithm finds a shortest path from a to z in the graph of Figure 1.

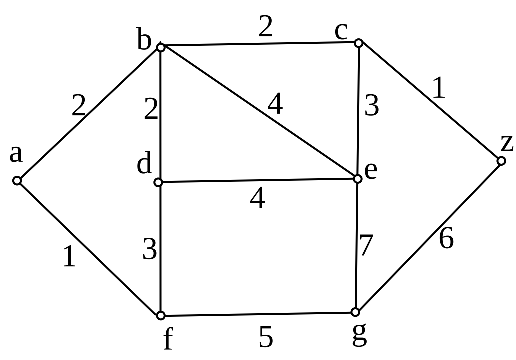


Figure 1

1. Put L(a) = 0. Figure 1 shows the result of executing line 1.

$T = \{a, b, c, d, e, f, g, z\}$

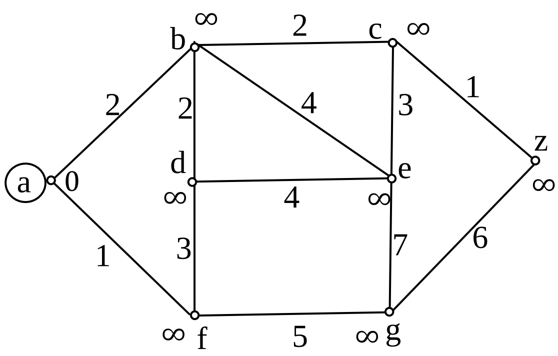


Figure 2

2. Since z is in T, according to line 2, we proceed to line 3.

3. We select vertex a, the uncircled vertex with smallest label, and circle it. Hence $T = \{b, c, d, e, f, g, z\}$

4. At line 4, we update each of the uncircled vertices b and f which are adjacent to a. We obtain the new labels. $L(b) = \min\{\infty, 0+2\} = 2$. $L(f) = \min\{\infty, 0+1\} = 1$. We return to line 2. Since $z \in T$, we proceed to line 3. We select vertex f and circle it. Now $T = \{b, c, d, e, g, z\}$

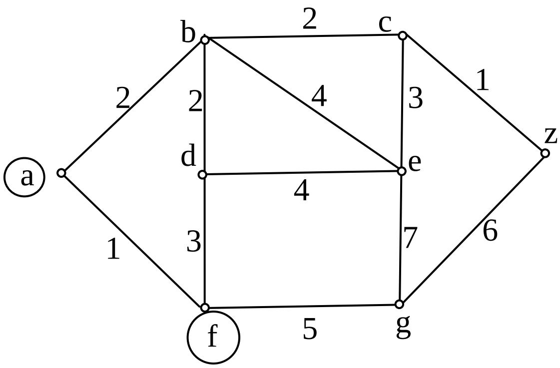


Figure 3

At line 4, we update the vertices d and g which are adjacent to f.

$$L(d) = \min\{\infty, 1+3\} = 4.$$

$$L(g) = \min\{\infty, 1+5\} = 6.$$

We return to line 2. Since $z \in T$, we proceed to line 3. We select vertex b with the smallest label and circle it. Now $T = \{c, d, e, g, z\}$.

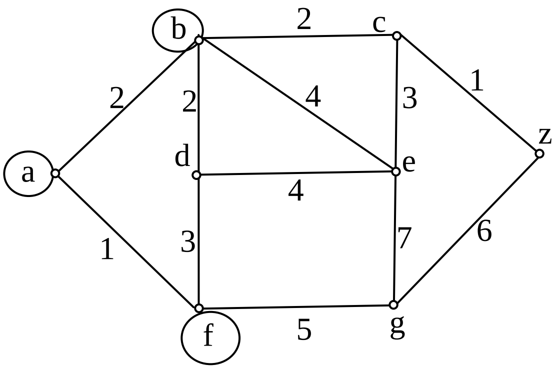


Figure 4

At line 4, we update the vertices c, d, e which are adjacent to b.

$$L(c) = \min\{\infty, 2+2\} = 4.$$

$$\mathrm{L(d)} = \min\{\infty,\ 2+2\} = 4.$$

$$\mathrm{L(e)} = \min\{\infty,\ 2+4\} = 6.$$

We return to line 2. Since $z \in T$, we proceed to line 3. We select vertex c with the smallest label and circle it. Now $T = \{d,\ e,\ g,\ z\}$.

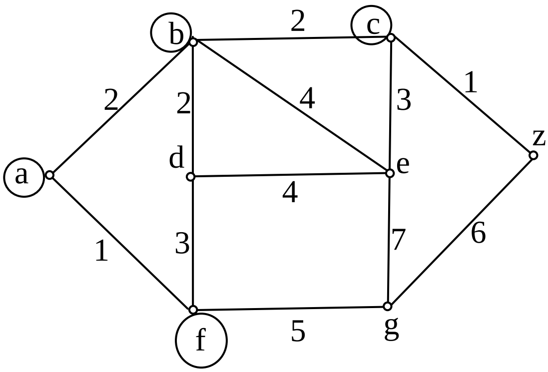


Figure 5

At line 4, we update the vertices e and z which are adjacent to c.

$$\mathrm{L(e)} = \min\{\infty,\ 4+3\} = 7.$$

$$\mathrm{L(z)} = \min\{\infty,\ 4+1\} = 5.$$

We select vertex z, with the smallest label and circle it.

Now T = { d, e, g }.

Since $z \notin T$, we stop.

L(z) = 6 is the length of shortest path from a to z.

The shortest path is (a, b, c, z).

Example. We show how the algorithm finds the length of a shortest path and a shortest path between each pair of vertices in the weighted graph.

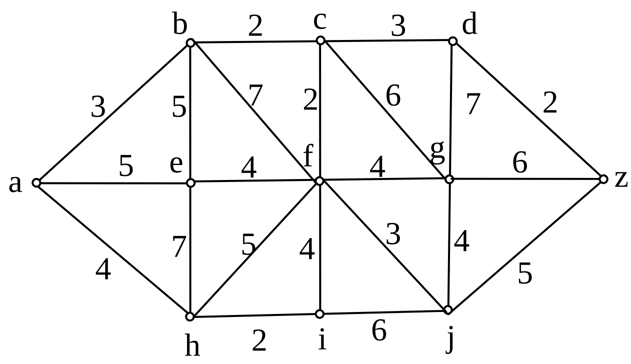


Figure 6

(i). Finding the shortest-path from a to f.

1. Put L(a) = 0, the Figure 6 shows the result of executing line 1.

$$T = \{a, b, c, d, e, f, g, h, i, j, z\}.$$

2. Since $f \in T$, according to line 2, we proceed to line 3.

3. We select vertex a, the uncircled vertex with smallest label and circle it. Hence $T = \{b, c, d, e, f, g, h, i, j, z\}$.

4. At line 4, we update each of the uncircled vertices b, e, h which are adjacent to a.

We obtain the new labels.

$$L(b) = \min\{\infty, 0+3\} = 3.$$

$$L(e) = \min\{\infty, 0+5\} = 5.$$

$$L(h) = \min\{\infty, 0+4\} = 4.$$

We return to line 2. Since $f \in T$, we proceed to line 3. We select vertex b and circle it.

Now $T = \{c, d, e, f, g, h, i, j, z\}$.

At line 4, we update the vertices c, e, f which are adjacent to b. We obtain the new labels.

$L(c) = \min\{\infty, 3+2\} = 5.$

$L(f) = \min\{\infty, 3+7\} = 10.$

$L(e) = \min\{\infty, 3+5\} = 8.$

We return to line 2. Since $f \in T$, we proceed to line 3. We select vertex h and circle it.

$T = \{c, d, e, f, g, i, j, z\}$.

At line 4, we update the vertices e, f, i which are adjacent to h.

$L(i) = \min\{\infty, 4+2\} = 6.$

$L(f) = \min\{\infty, 4+5\} = 9.$

$L(e) = \min\{\infty, 4+7\} = 11.$

We return to line 2. Since $f \in T$, we proceed to line 3. We select vertex c and circle it.

$T = \{d, e, f, g, i, j, z\}$.

At line 4, we update the vertices f, g, d which are adjacent to c.

$L(f) = \min\{\infty, 5+2\} = 7.$

$L(g) = \min\{\infty, 5+6\} = 11.$

$L(d) = \min\{\infty, 5+3\} = 8.$

We return to line 2. Since $f \in T$, we proceed to line 3. We select vertex f with the smallest label and circle it. Now, $T = \{d, e, g, i, j, z\}$.

Since $f \notin T$, we stop. L(f) = 7 is the length of shortest path from a to f. The shortest path is (a, b, c, f).

(ii). Finding the shortest-path from b to j.

1. Put L(b) = 0. The Figure 6 shows the result of executing line 1.

$T = \{b, c, d, e, f, g, h, i, j, z, a\}$.

2. Since $j \in T$, according to line 2, we proceed to line 3.
3. We select vertex b, with the smallest label and circle it.

Hence $T = \{c, d, e, f, g, h, i, j, z, a\}$.

4. At line 4, we update each of the uncircled vertices a, e, f, c which are adjacent to b. We obtain the new labels.

$$L(a) = \min\{\infty, 0+3\} = 3.$$

$$L(e) = \min\{\infty, 0+5\} = 5.$$

$$L(f) = \min\{\infty, 0+7\} = 7.$$

$$L(c) = \min\{\infty, 0+2\} = 2.$$

We return to line 2. Since $j \in T$, we proceed to line 3. We select vertex c and circle it.

Now $T = \{d, e, f, g, h, i, j, z, a\}$.

At line 4, we update the vertices f, g, d which are adjacent to c.

We obtain a new label.

$$L(f) = \min\{\infty, 2+2\} = 4.$$

$L(d) = \min\{\infty, 2+3\} = 5.$

$L(g) = \min\{\infty, 2+6\} = 8.$

We return to line 2. Since $j \in T$, we proceed to line 3. We select vertex a with smallest label and circle it. $T = \{d, e, f, g, h, i, j, z\}$.

At line 4, we update the vertices e and h which are adjacent to a.

$L(e) = \min\{\infty, 3+5\} = 8.$

$L(h) = \min\{\infty, 3+4\} = 7.$

We return to line 2. Since $j \in T$, we proceed to line 3. We select vertex f with smallest label and circle it.

$T = \{d, e, g, h, i, j, z\}$.

At line 4, we update the vertices e, h, i, j and g which are adjacent to f.

$L(i) = \min\{\infty, 4+4\} = 8.$

$L(j) = \min\{\infty, 4+3\} = 7.$

$L(g) = \min\{\infty, 4+4\} = 8.$

We return to line 2. Since $j \in T$, we proceed to line 3. We select vertex d with the smallest label and circle it.

Now $T = \{e, g, h, i, j, z\}$.

At line 4, we update the vertices g and z which are adjacent to d.

$L(g) = \min\{\infty, 5+7\} = 12.$

$L(z) = \min\{\infty, 5+2\} = 7.$

We return to line 2. Since $j \in T$, we proceed to line 3. We select vertex j with the smallest label and circle it.

Now $T = \{e, g, h, i, z\}$.

Since $j \notin T$, we stop. $L(j) = 7$ is the length of shortest path from b to j.

Hence, the shortest path is (b, c, f , j).

Application. We show how the algorithm finds a shortest path from Wundwin to Thit Sone Gyi in the graph of Figure 7.

1. Put L(Wundwin) = 0, the Figure 7 shows the result of executing line 1.

   T = {Wundwin, ThaPhayWa, Meiktila, Thazi, Payangazu, Pyawbwe, Yamethin, Hkanbu, Thit Sone Gyi, Ah Le Ywa, Shan Ma Nge, Sedo, Kyauk Tan, Mahlaing }.

2. Since Thit Sone Gyi $\in T$, according to line 2, we proceed to line 3.

3. We select Wundwin, the uncircled vertex with smallest label and circle it.

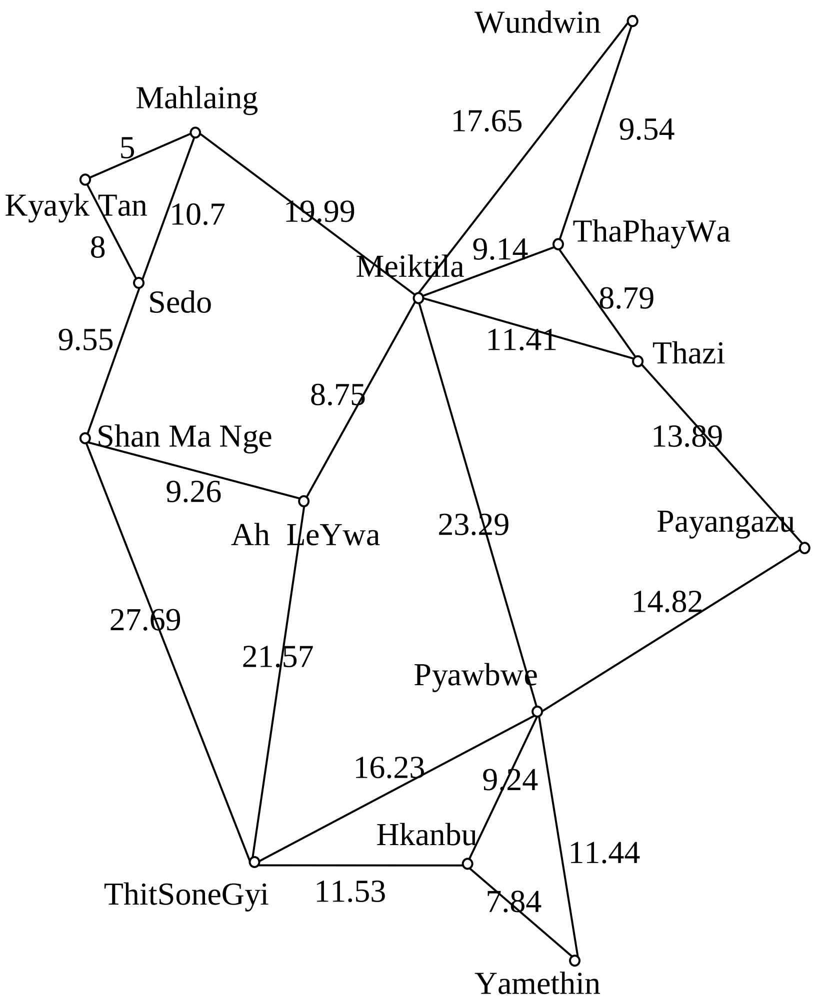


Figure 7

Hence T={Tha Phay Wa, Meiktila, Thazi, Payangazu, Pyawbwe, Yamethin, Hkanbu, Thit Sone Gyi, Ah Le Ywa, Shan Ma Nge, Sedo, Kyauk Tan, Mahlaing }.

4. At line 4, we update each of the uncircled Tha Phay Wa and Meiktila which are adjacent to Wundwin.

We obtain the new labels.

$L(\text{Tha Phay Wa}) = \min\{\infty,\ 0+9.54\} = 9.54$ miles.

$L(\text{Meiktila}) = \min\{\infty,\ 0+17.65\} = 17.65$ miles.

We return to line 2. Since $\text{Thit Sone Gyi} \in T$, we proceed to line 3. We select Tha Phay Wa and circle it.

Hence T = {Meiktila, Thazi, Payangazu, Pyawbwe, Yamethin, Hkanbu, Thit Sone Gyi, Ah Le Ywa, Shan Ma Nge, Sedo, Kyauk Tan, Mahlaing }.

At line 4, we update Meiktila and Thazi which are adjacent to Tha Phay Wa.

We obtain the new labels.

$L(\text{Meiktila}) = \min\{\infty,\ 9.54+9.14\} = 18.68$ miles.

$L(\text{Thazi}) = \min\{\infty,\ 9.54+8.79\} = 18.33$ miles.

We return to line 2. Since $\text{Thit Sone Gyi} \in T$, we proceed to line 3. We select Meiktila and circle it.

Now T = {Thazi, Payangazu, Pyawbwe, Yamethin, Hkanbu, Thit Sone Gyi, Ah Le Ywa, Shan Ma Nge, Sedo, Kyauk Tan, Mahlaing}.

At line 4, we update the Mahlaing, Ah Le Ywa, Pyawbwe and Thazi which are adjacent to Meiktila.

We obtain the new labels.

$L(\text{Mahlaing}) = \min\{\infty, 17.65+19.99\} = 37.64$ miles.

$L(\text{Ah Le Ywa}) = \min\{\infty, 17.65+8.75\} = 26.4$ miles.

$L(\text{Pyawbwe}) = \min\{\infty,\ 17.65+23.29\} = 40.94$ miles.

$L(\text{Thazi}) = \min\{\infty,\ 17.65+11.41\} = 29.06$ miles.

We return to line 2. Since, $\text{Thit Sone Gyi} \in T$, we proceed to line 3. We select Ah Le Ywa with the smallest label and circle it.

Hence T = {Thazi, Payangazu, Pyawbwe, Yamethin, Hkanbu,Thit Sone Gyi, Shan Ma Nge, Sedo, Kyauk Tan, Mahlaing }.

At line 4, we update Shan Ma Nge, and Thit Sone Gyi which are adjacent to Ah Le Ywa.

We obtain the new labels.

$$L(\text{Shan Ma Nge}) = \min\left\{\infty,\ 26.4 + 9.26\right\} = 35.66\ \text{miles.}$$

$$L(\text{Thit Sone Gyi}) = \min\left\{\infty,\ 26.4 + 21.57\right\} = 47.97\ \text{miles.}$$

We select Thit Sone Gyi, and circle it.

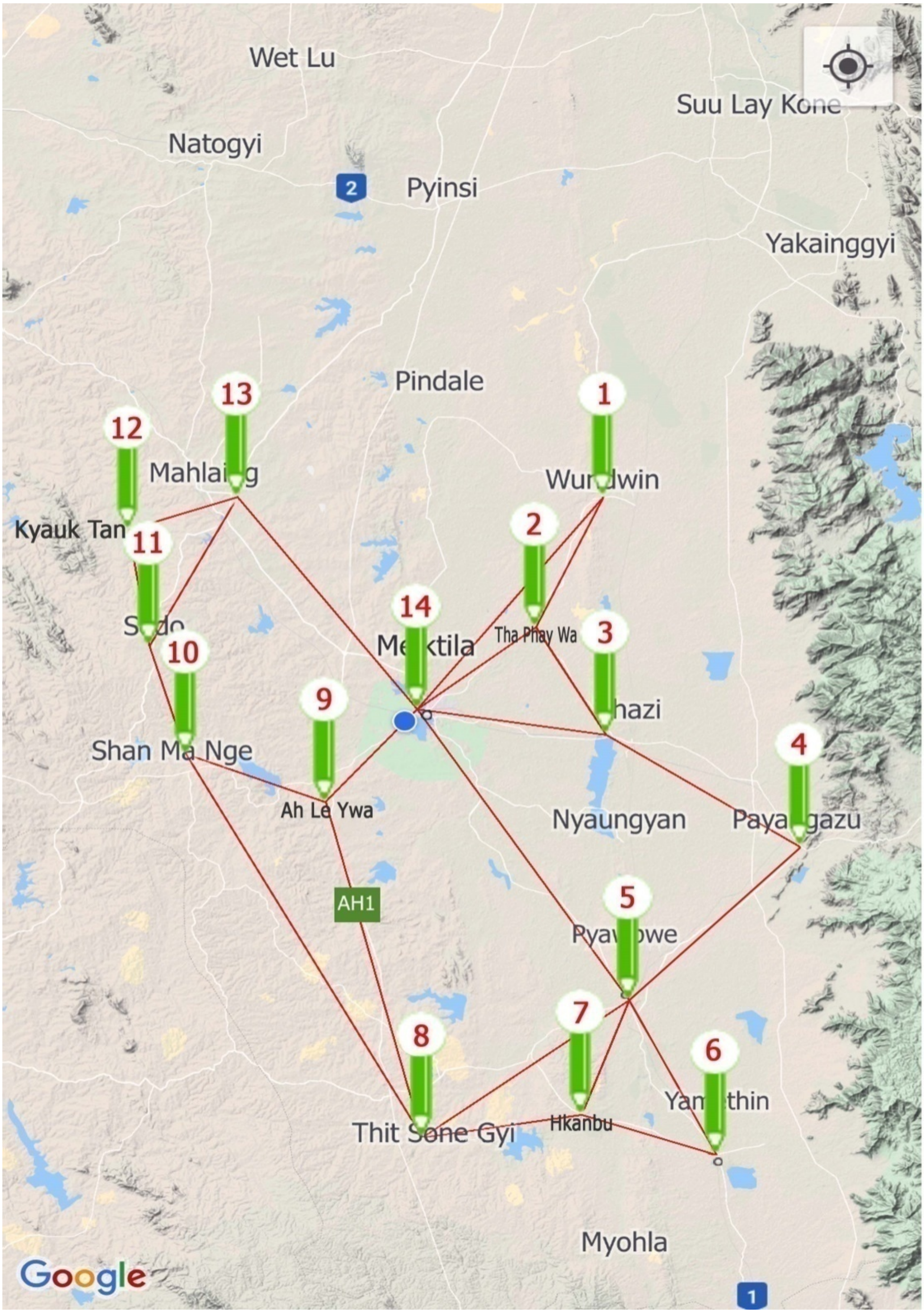
Now, T = {Thazi, Payangazu, Pyawbwe, Yamethin, Hkanbu, Shan Ma Nge, Sedo, Kyauk Tan, Mahlaing }.

Since, $\text{Thit Sone Gyi} \notin T$, we stop.

L(Thit Sone Gyi) = 47.97 miles is the length of shortest path from Wundwin to Thit Sone Gyi.

The shortest path (Wundwin, Meiktila, Ah Le Ywa, Thit Sone Gyi).

Thus, the shortest- path from Wundwin to Thit Sone Gyi in this graph is 47.97 miles


Wet Lu
Suu Lay Kone
Natogyi
Pyinsi
2
Yakainggyi
Pindale
13
12
1
Mahlai g
Wur dwin
Kyauk Tan
11
2
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3
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Tha Phay Wa
9
hazi
4
Shan Ma Nge
Ah Le Ywa
Nyaungyan
Paya gazu
AH1
5
7
8
6
Hkanbu
Yam thin
Thit Sone Gyi
Myohla
Google
1

Conclusion

Dijkstra's Shortest-Path Algorithm is presented and considered with examples. Then finding the shortest -path between two Locations are studied.

Acknowledgements

I would like to express my grateful thanks to Head and Professor, Dr San San Wai and Professor, Dr Khin Myat Myat Aung, Department of Mathematics, Meiktila University for their hearty attention, advice and encouragement. I also thank all my colleagues of our department for their warm friendship and kind advices.

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